Loran-C Time Difference Control Using Kalman Filtering

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Abstract

Despite oscillator improvements, in order for the Loran-C signal to remain within specifications, a time difference control (TDC) algorithm must be applied. Currently TDC is being performed using a linear least squares algorithm. This paper explores the use of Kalman filtering as a way to predict future timing error to determine the optimal phase adjustment to be made. The use of a controller consisting of Kalman filter predictions and a discrete linear quadratic regulator (LQR) to solve for control inputs does not consistently improve system performance. The estimate of the states is much better using Kalman filtering compared to least squares. Using Kalman filtering with a different controller to solve the cost function (other than LQR) and further "tuning" the Kalman filter would probably improve system performance.

Introduction

Loran-C is a radionavigation system that enables a position to be calculated based on the time difference of signal arrival from several towers in a chain. Each tower provides a parabolic line of position based on the signals time of arrival. It is currently used in several applications, including marine navigation, as a timing source, aeronautical navigation, and in the future landing of aircraft. This system has a very high repeatability. To improve Loran's performance this project explores alternate time difference control algorithms. USCG Loran Support Unit sponsored this project.

Background

The Loran-C system is set up into chains. Each chain has a master transmitter and at least two secondary transmitters. The master sends out a pulse and each secondary transmits a pulse at a designated time after the master. A Loran fix is computed based on the time difference of arrival (TD) of a master-secondary pair. Each master-secondary pair TD gives a hyperbolic line of position.

Currently Loran timing is being controlled by the Loran Consolidated Control System (LCCS). LCCS receives TD data from monitoring stations. These monitoring stations are several hundred miles away from the transmitter. There are several monitoring stations (also know as PKMS sights) in every Loran-C chain. The monitoring station closest to the transmitter is designated the Alpha 1 monitor. The next closest monitor is designated Alpha 2. This provides redundancy in case of a failure. LCCS uses the TD data received from the monitoring stations to control the time difference, and hence the accuracy of Loran.

The TD received would be constant without oscillator drift or atmospheric and other noise. A control standard time difference (CSTD) has been established at every monitoring station for every transmitter monitored by that sight. Each monitoring site transmitter pair is called a

baseline. The deviation from the CSTD is the time difference error (TDE). The cumulative TDE is a measure of error over time.

To control the system, LCCS monitors the TDE and the cumulative TDE. When either of these values are not within tolerances, it sends a local phase adjustment (LPA) to the transmitter. The method used to keep Loran within specification is called Loran time difference control (TDC).

There are three competing factors that LCCS takes into account when deciding when and the amount of LPA to be added in (present TDE, cumulative TDE, and LPA). The three competing factors are to keep present TDE within plus and minus one half the tolerance bound (100 nanoseconds) of the CSTD, keep cumulative TDE as close to zero as possible, and minimize the size and number of LPA's added.

Controlling TD is a balancing act; there are political, policy, and functionality concerns. For example, the Coast Guard has unwritten agreements with the Massachusetts Lobstermen's Association to keep cumulative TDE low. Cumulative TDE effects the repeatability of Loran. Lobstermen and fishermen want repeatability so that they can mark good fishing spots and find gear/traps left out. Navigational users want present TDE to be minimal because that affects Loran's absolute accuracy. Absolute accuracy is how close Loran tells you are compared to where you actually are. When you minimize one of these factors you negatively affect the others. Minimizing present TDE improves the accuracy, but at the cost of using more LPA's and possibly allowing cumulative TDE to drift. Minimizing cumulative TDE improves the repeatability, but at the cost of adding more LPA's and allowing present TDE to drift. Minimizing the number and size of LPA's causes users to not see jumps in position, but at the cost of allowing cumulative and present TDE drift.

Researchers at Naval Postgraduate School Monterey, CA developed a Kalman filter controller for Loran time difference control. This research is an extension of that work.

Objective

The goal of this project is to determine the best method to control the accuracy of Loran to meet and exceed signal specifications while putting in the least number of LPA's. Methods that will be looked at are using no control algorithm, the present controller (LCCS), and using Kalman filtering. To accomplish this goal we need to establish a system model for insertion of LPA's, remove LCCS's effects on the stored data, and develop a Kalman filter.

Theory

LCCS estimates future TDE and cumulative TDE values using curve fitting. To predict future TDE and cumulative TDE, LCCS reads in 90 minutes of data. The first 45 minutes are fit to a third-order polynomial, a linear least squares estimate of TDE. The midpoint of this polynomial fitting is the predicted value or present TDE.

Equation 1 is the cost function that LCCS is solving.

$$J = \frac{f_1(TDE,u)}{k_s^2} + \frac{f_2(EP,u)}{k_i^2} + f_3(u) \quad \text{(Eq 1)}$$

Where f_1 represents the predicted TDE, f_2 represents the cumulative TDE, and f_3 is a similar function that represents LPA [4]. Based on this prediction the LPA is calculated by substituting nine different values for LPA (from –80ns to 80ns in 20ns increments) into the cost function. The LPA that produces the lowest cost is recommended.

In contrast, we are using Kalman filtering to estimate future TDE's and cumulative TDE's. Then based upon this predicted information, the optimal LPA is found using the discrete linear quadratic regulator solution.

The Kalman filter estimates future states of a system that is affected by noise based on assumed statistical properties of the noise. It is a computer algorithm and not really a filter, in the sense that it does not give a desired frequency response. The Kalman filter has many advantages. It is a recursive algorithm that takes into account all past system information [1]. The filter estimates future states in the presence of noise even when the exact workings of system being modeled is unknown.

The Kalman Filter equations model the system being controlled using two equations:

$$X_k = A * X_{k-1} + B * U_k + w_{k-1} \quad \text{(Eq 2)}$$
$$Z_k = H * X_k + v_k \quad \text{(Eq 3)}$$

Equation 2 is the process equation and Equation 3 is the measurement, or observation equation, where X_k is the process state matrix (m x 1) at sample time k. There are m states to the system. A is a (m x m) matrix that relates X_k to X_{k-1}. U_k is the system input(s). B is a (m x 1) matrix relates U_k to X_k. w_{k-1} is a (m x 1) matrix that represents the process noise that has as known covariance structure, Z_k is a (m x 1) measurement vector at time k, H is a (m x m) matrix giving the noiseless connection between the measurement and the state vector at time k, and v_k is a (m x 1) vector of the measurement noise, assumed to be a white sequence with known covariance structure and having zero crosscorrelation (independent) with the w_k sequence [1].

The two sets of equations that the Kalman Filter uses to predict states are the time update equations and the measurement update equations.

The time update equations are:

$$\hat{X}_k^- = A * \hat{X}_{k-1} + B * U_k \quad \text{(Eq 4)}$$
$$P_k^- = A * P_{k-1} * A^T + Q_k \quad \text{(Eq 5)}$$

Where K_k is the Kalman gain. $\hat{X}_k$ is the estimated state after the measurement (*a posteriori*), and $\hat{X}_k^-$ is the *a priori* (before the measurement) estimate of the state. The "hat" denotes an estimate and the "super minus" reminds us that this is our best estimate of the state before using the measurement at k. P_k is the error covariance matrix. Q_k is the process noise covariance, and R_k is the measurement noise covariance. These values can change with time, but often they are assumed to be constant.

The first of the Kalman time update equations (Eq 4) predicts the next state in the absence of a measurement. The second Kalman time update equation (Eq 5) updates the P matrix.

The measurement equations are:

$$K_k = \frac{P_k^- * H^T}{H * P_k^- * H^T + R_k} \quad \text{(Eq 6)}$$

$$\hat{X}_k = \hat{X}_k^- + K_k * (Z_k - H_k * \hat{X}_k^-) \quad \text{(Eq 7)}$$

$$P_k = (I - K_k H) * P_k^- \quad \text{(Eq 8)}$$

The first measurement update equation computes the Kalman gain (Equation 6). The Kalman gain is calculated to minimize the mean square error between the actual state and the estimated state. The next equation (Equation 7) in the measurement update equations updates the *a priori* estimate with a measurement. The final measurement update equation (Equation 8) updates the error covariance matrix P.

The entire reiterative Kalman filter process is shown in figure 1.

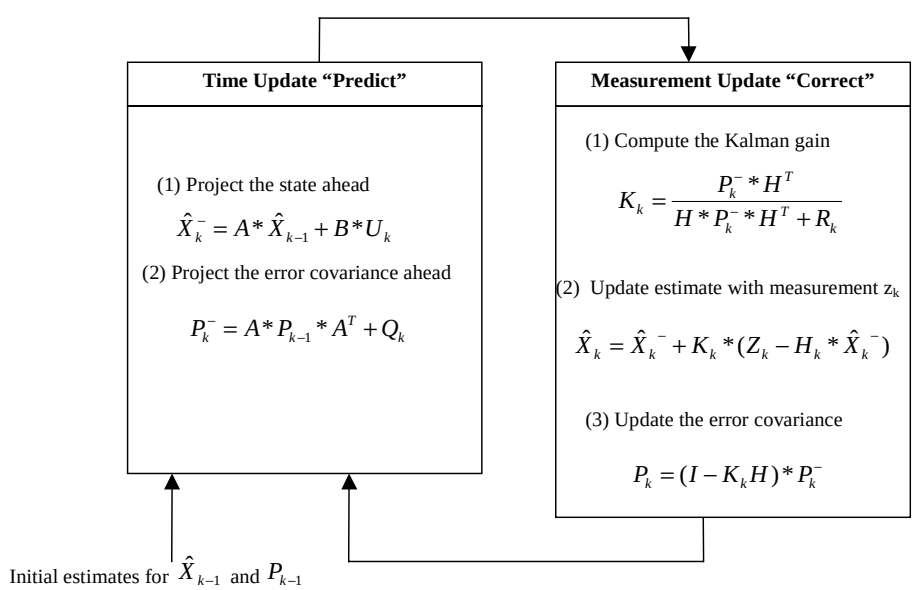

Figure 1 [3]

Now we must make the general Kalman equations specific for Loran time difference control. The system can be defined using three states the time difference error (TDE), the rate of change of TDE ($T\dot{D}E$), and the cumulative TDE (cum_TDE). The system input is the local phase adjustment (LPA). The process equation becomes:

$$\underbrace{\begin{bmatrix} \mathrm{TDE}(k) \\ \mathrm{T\dot{D}E} \\ \mathrm{cum_TDE}(k) \end{bmatrix}}_{X_k} = \underbrace{\begin{bmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 1 & T & 1 \end{bmatrix}}_{A} * \underbrace{\begin{bmatrix} \mathrm{TDE}(k-1) \\ \mathrm{T\dot{D}E(k-1)} \\ \mathrm{cum_TDE}(k-1) \end{bmatrix}}_{X_{k-1}} + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}_{B} * \underbrace{\Delta\Phi(k-1)}_{U_{k-1}} \quad \text{(Eq 9)}$$

We make three "measurements": TDE, average of differences of the past three TDE's (diff_TDE) (this helps to smooth the slope between two points that would be large after the addition of an LPA {20ns/7.5min}), and cumulative TDE. Present TDE is collected as data; then cumulative TDE and the TDE difference are calculated from the data point. The measurement equation becomes:

$$\underbrace{\begin{bmatrix} \mathrm{TDE}(k) \\ \mathrm{diff_TDE}(k) \\ \mathrm{cum_TDE}(k) \end{bmatrix}}_{Z_k} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{H} * \underbrace{\begin{bmatrix} \mathrm{TDE}(k) \\ \mathrm{T\dot{D}E\ (k)} \\ \mathrm{cum_TDE}(k) \end{bmatrix}}_{X_k} \quad \text{(Eq 10)}$$

where T is the sampling period, 7.5 minutes.

We determine LPA's by using the dlqr (L=dlqr(A,B,Q,R)) command in MATLAB. This function finds L, which minimizes:

$$J = \sum_{n=0}^{\infty} X^T Q X \ + U^T R\, U \qquad \text{(Eq 11)}$$

Where:

$$U = -L * X \qquad \text{(Eq 12)}$$

and

$$X = A * X_{k-1} + B * U_{k-1} \qquad \text{(Eq 13)}$$

Q and R are weighting matrices that indicate the relative cost of X and U.

For our problem

$$\underbrace{\begin{bmatrix} \mathrm{TDE(k)} \\ \mathrm{cum_TDE(k)} \end{bmatrix}}_{X_k} = \underbrace{\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}_{A} * \underbrace{\begin{bmatrix} \mathrm{TDE(k-1)} \\ \mathrm{cum_TDE(k-1)} \end{bmatrix}}_{X_{k-1}} + \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{B} * \underbrace{\Delta\Phi(k-1)}_{U_{k-1}} \quad \text{(Eq 14)}$$

And the cost function is

$$J(k) = \sum_{-\infty}^{\infty} \{ q_1 \left| \mathrm{TDE}(k) \right|^2 + q_2 \left| \mathrm{cum_TDE}(k) \right|^2 + R * \left| \Delta\Phi_k \right|^2 \} \quad \text{(Eq 15)}$$

Relating Equation 15 to Equation 13:

$$J = \sum_{n=0}^{\infty} \begin{bmatrix} \mathrm{TDE}(k) \\ \mathrm{cum_TDE}(k) \end{bmatrix} * \begin{bmatrix} \mathrm{q}_1 & 0 \\ 0 & \mathrm{q}_2 \end{bmatrix} * \begin{bmatrix} \mathrm{TDE}(k) \\ \mathrm{cum_TDE}(k) \end{bmatrix} + \Delta\Phi * R * \Delta\Phi \quad \text{(Eq 16)}$$

Technical Approach

To test and implement this algorithm, data was gathered from Loran Support Unit and then processed. First, the TD data was converted to TDE data. Next, the effects of LCCS were removed by removing the LPA's. This simulates what would have happened if no controller was used. Once this raw data was prepared, a Kalman filter algorithm was developed and then tuned. The results were then analyzed.

TD data was extracted from the database at USCG Loran Support Unit for the transmitters in two different chains (USWC and NOCUS) from 01 February 2001 to 13 March 2001. The TD data received was in 10-second samples[1]. Since the system is prone to noise, LCCS reads in 45 of these samples (for a total of 7.5 minutes of data) and then averages them. Our system does the same averaging

For LCCS, Cumulative TDE was calculated by inputting 90 minutes of data. The first 45 minutes was used to "predict" the next 45 minutes of data. The first 45 minutes of data was put through a smoothing function and the midpoint of the smoothed function was the TDE value that was used to update the cumulative TDE. For the Kalman algorithm, cumulative TDE was updated using the present TDE.

The receivers at these monitoring stations have recently been replaced (as will all receivers). They do 60-second averages of real-time data (NOTE: the time spent average can be different for every baseline). Figure 2 is a plot of the receiver's outputted signal's response to a 20-nanosecond input.

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[1] The data received was TD data given in nanoseconds. Giving the TD in just nanoseconds assumes that the TD is the correct CSTD to the tens of nanoseconds. This assumption was not always correct. When the TD vector was plotted, there was discontinuity, in that there were TD points near the x-axis and then another set around 100. This shows that the CSTD was assumed to one too many significant figures. Each data set had to use a different test and correct program to fix this error.

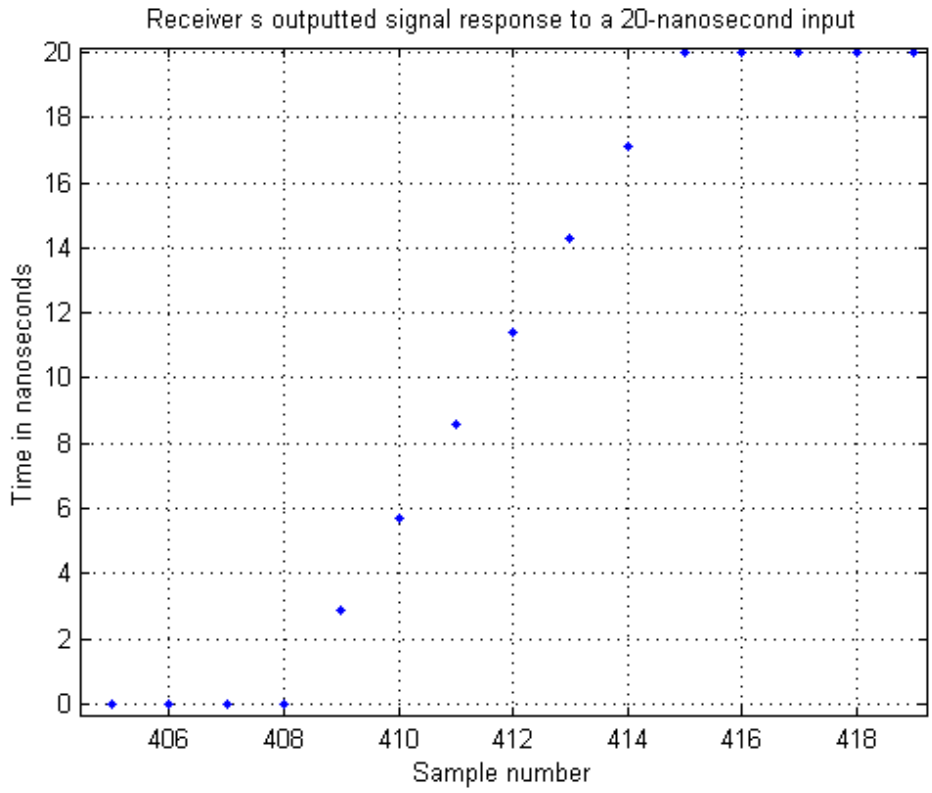


Figure 2

Note the linear response. Since it takes six samples (when sampling rate is 10 sec) for the system to fully respond to the input and the response is linear, each sample is affected increasingly by 1/6 time the input (until the value reaches 6/6, then 1 time the input for the rest of the sample). This response has been modeled when removing LPA's during unprocessing and inserting LPA's during the Kalman simulation.

Results

Searchlight and Baudette alpha-1 baselines were the two data sets used for this analysis. The data sets consisted of Alpha-1 data from 01FEB01-15MAR01.

Data sets were plotted "as-is." These plots are what actually happened with LCCS doing time difference control (Figures 5-7). The LPA's were removed and the data was plotted. These plots were what would have happened if no controller was used. The data after having the LPA's removed was then passed to our Kalman filter/LQR controller. The weighting factors for the LQR (R_{lqr}, q_1, and q_2) were varied to try and produce the best result.

First, let's take a look at what happens if we used no control algorithm at all. Figure 3 is a plot of Searchlight TDE data over the entire 43 days after removal of the LPA's added in by LCCS.

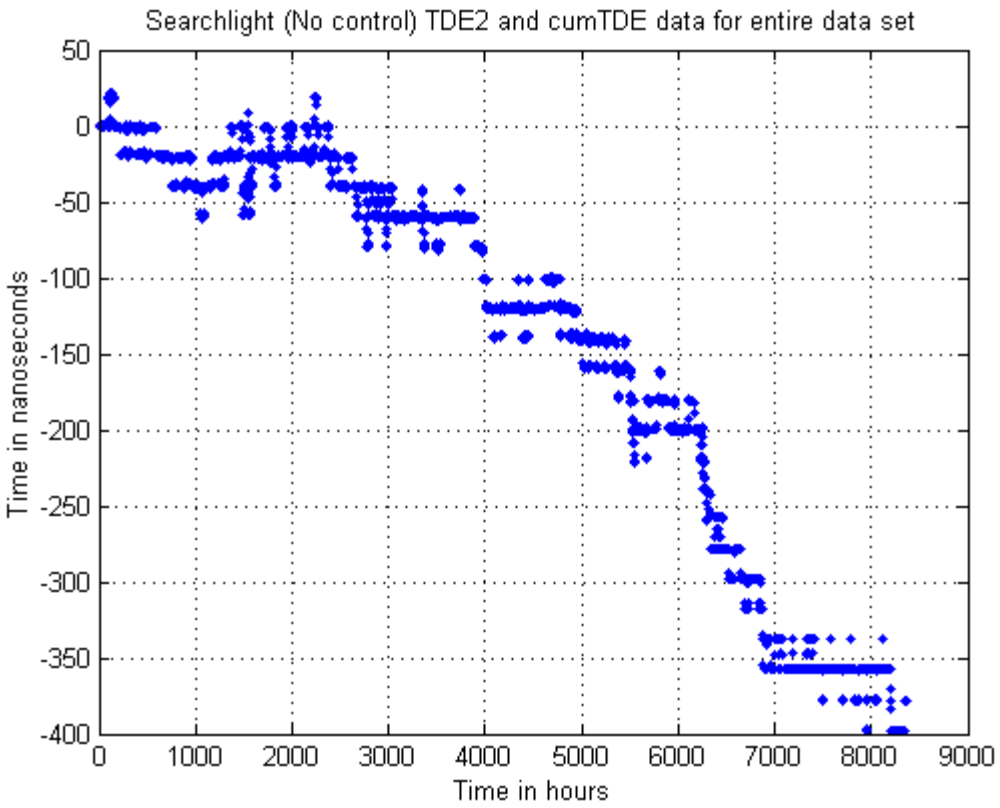


Figure 3

Clearly the present TDE does not fall within the plus or minus 100ns specifications. Cumulative TDE was not plotted because it exponentially decreases and would make the scale of this graph too small. Notice the negative trend in the present TDE that could be caused by oscillator drift. Now let's look at Baudette with no control (Figure 4).

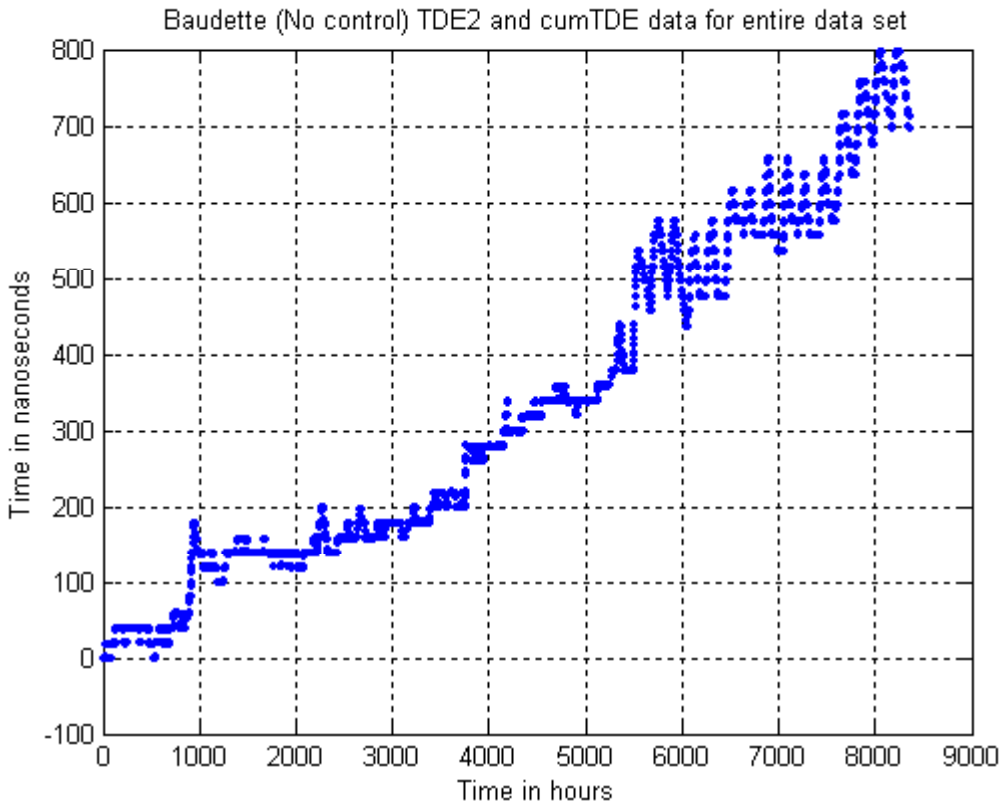
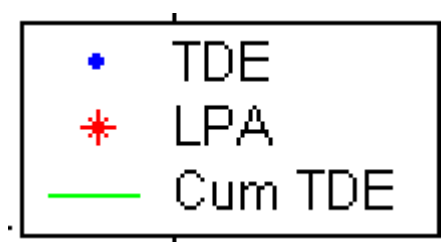


Figure 4

Notice that there is a general drift upwards. This could also be due to oscillator drift. In the segment beginning at 5500, besides general oscillator drift upward, there is also a drift downwards. This downward drift could be propagation effects caused by weather since Baudette is in northern Minnesota and the data is taken during February and March.

Let's see how LCCS did with controlling time difference (Figures 5-7).

The legend for all plots is:



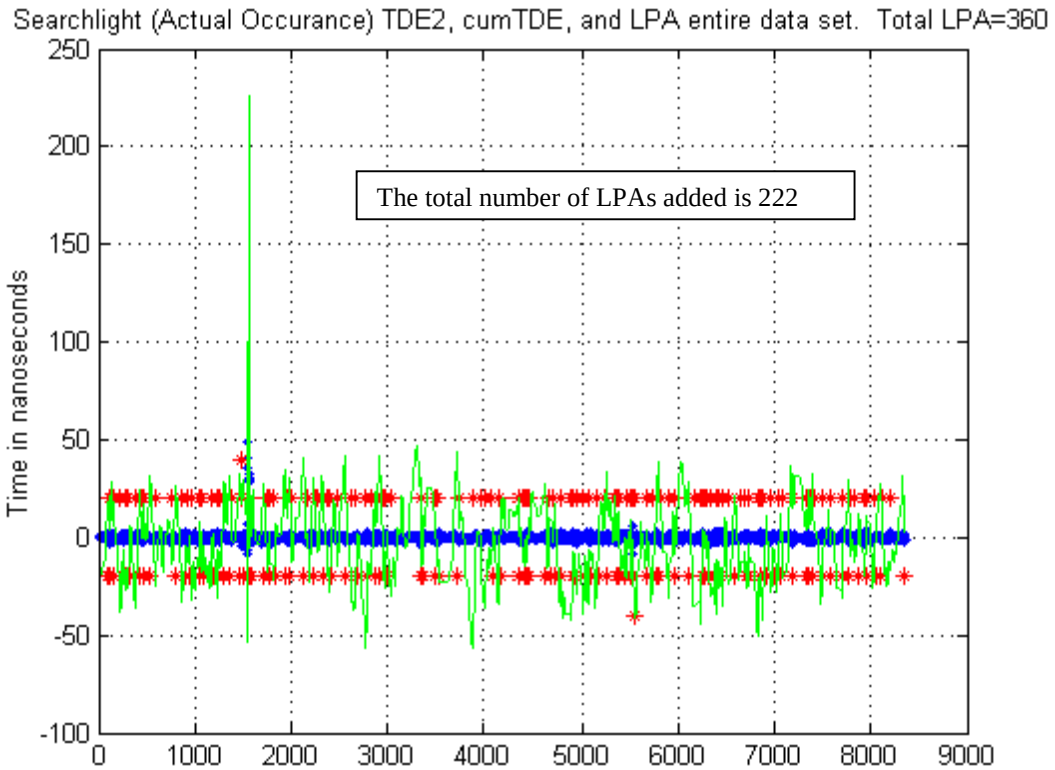


Figure 5

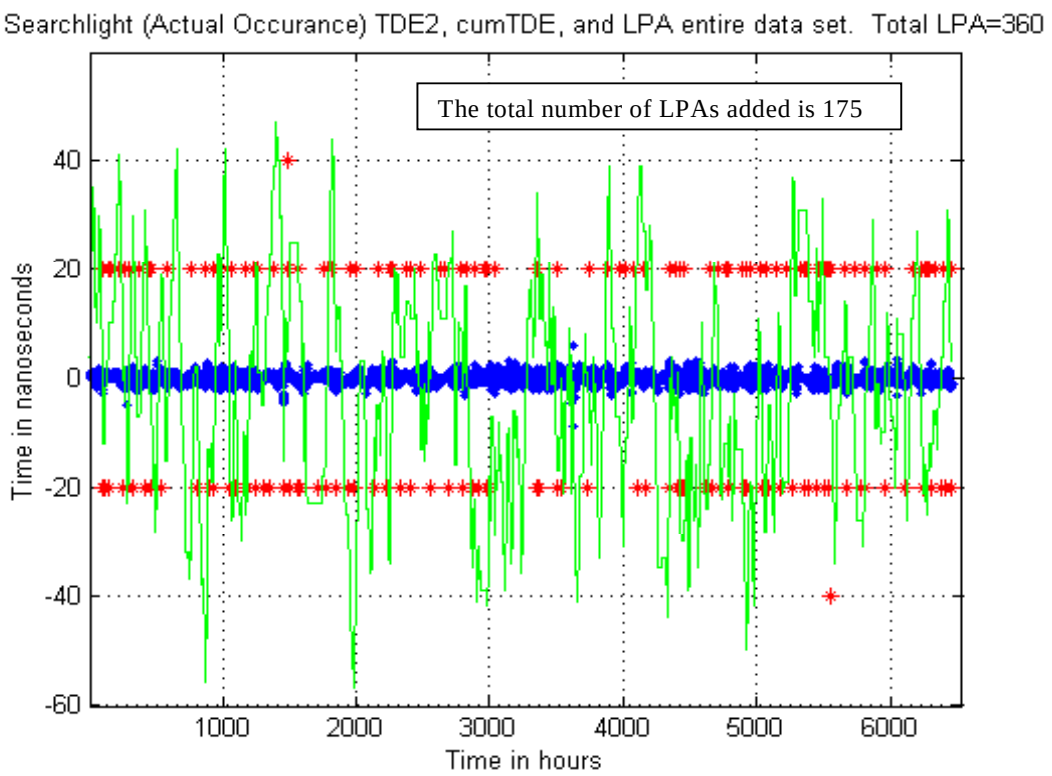


Figure 6

Due to the spike at sample 1800 (Figure 5), Searchlight data was only used from sample 1900 on (Figure 6).

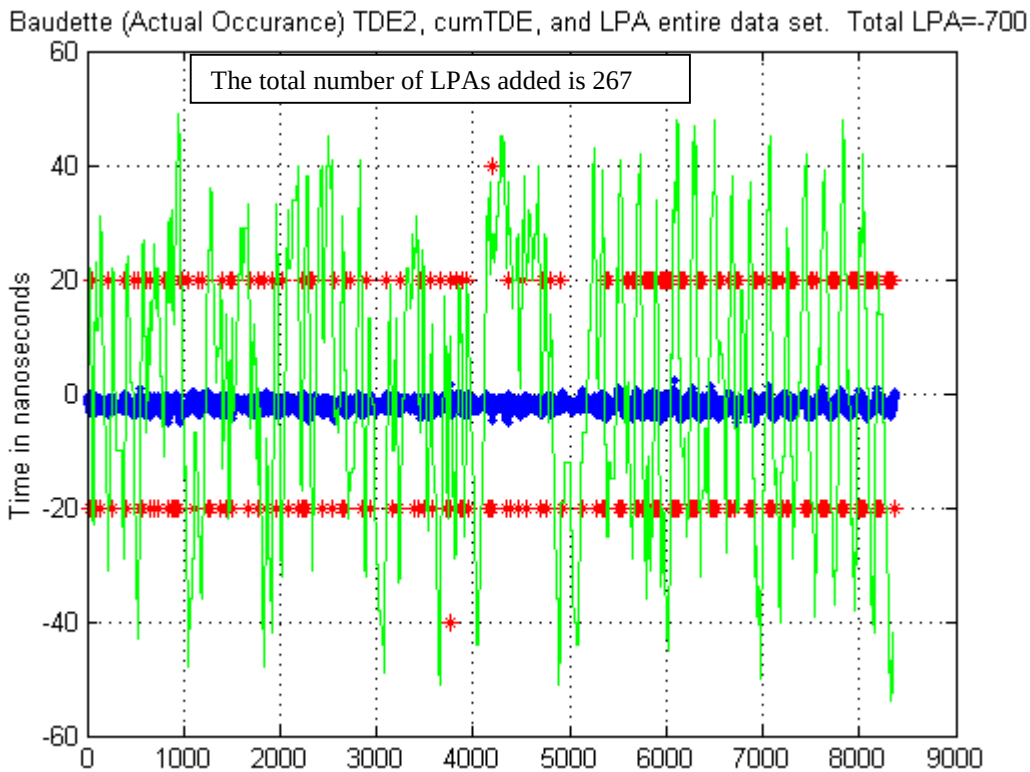


Figure 7

LCCS keeps cumulative TDE within 55 nanoseconds of zero and present TDE within 10 nanoseconds of zero. The present TDE and cumulative TDE on the Searchlight plots spike on one day but quickly recover.

The first set of weightings used in the Kalman filter was $R_{lqr} = 1$, $q_1 = \frac{1}{20^2}$, and $q_2 = \frac{1}{20^2}$. These values were used because LCCS was using values of k_i and k_s equal to 20. This weights the LWR cost function the same as the cost function used by LCCS (Figures 8-9).

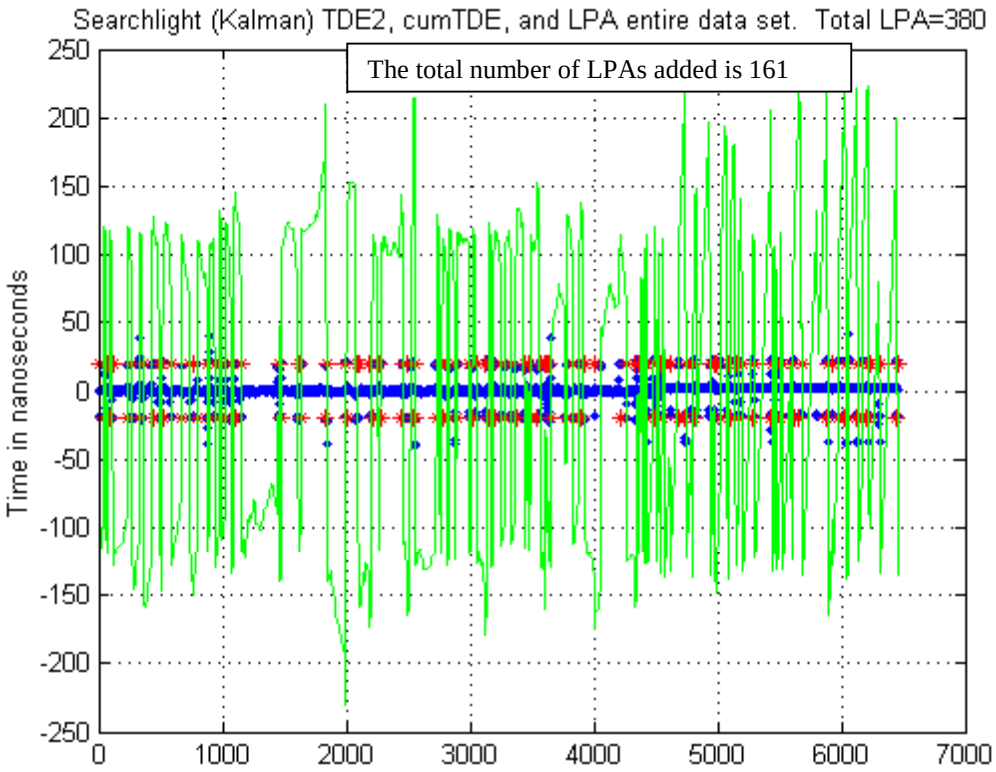


Figure 8

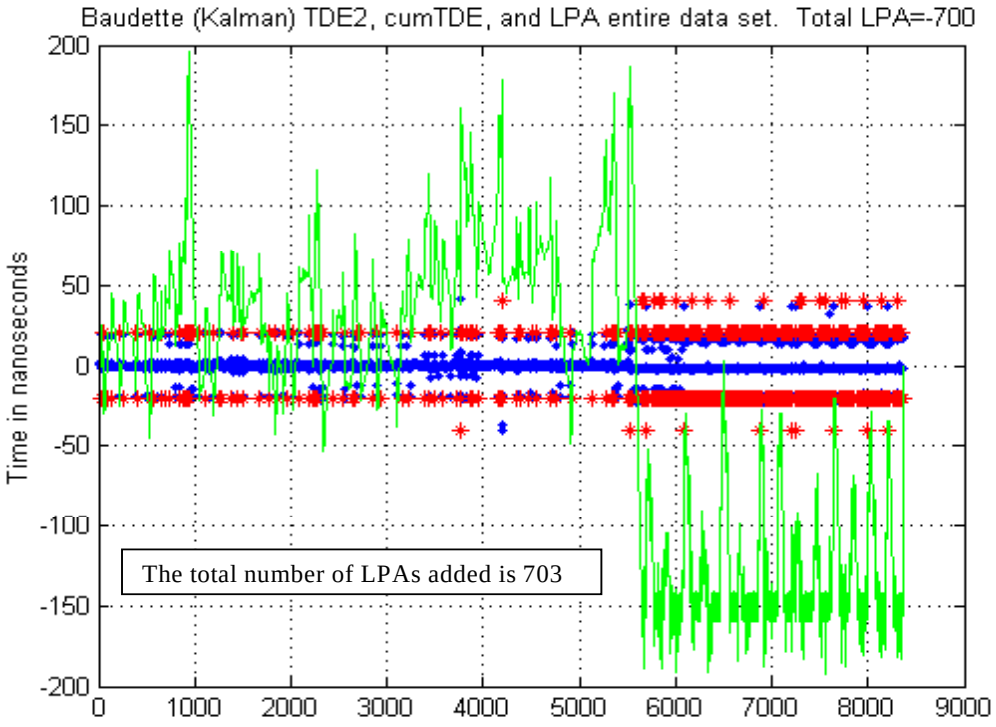


Figure 9

For both Searchlight and Baudette, the number of LPA's added in was slightly lower than LCCS, but this came with the cost of allowing cumulative TDE to vary greatly (more so than it did while LCCS was doing control). The Kalman/LQR controller also allowed TDE to vary greatly between +20ns and –20ns. This set of weighting put the highest cost on the number and size of LPA's

Let's apply the same amount of cost to cumulative TDE . The weighting factors are $R_{lqr} = \frac{1}{20^2}$, $q_1 = \frac{1}{20^2}$, and $q_2 = 1$ (Figure 10).

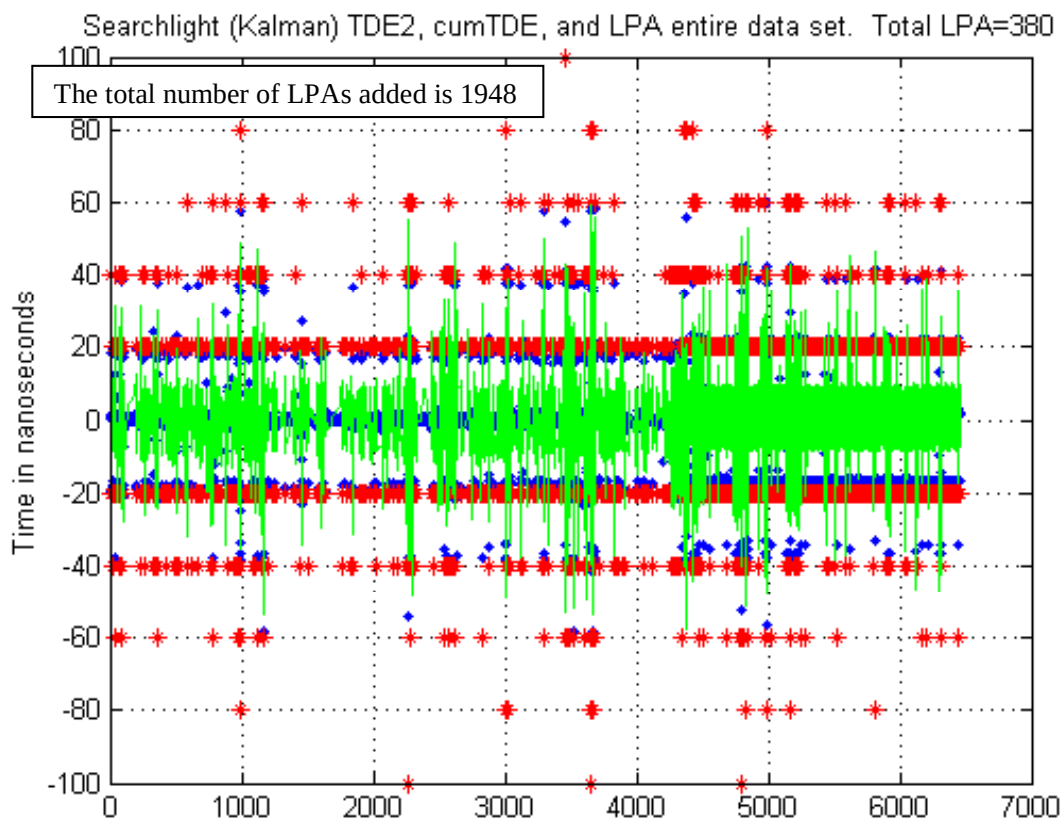


Figure 10

These weighting factors keep cumulative TDE within –60ns to +60 ns. The trade off is the high number of LPA's and some TDE values off by 60ns. The values were varied to try and come up with the best solution.

Figure 11 has the scale factors of $R_{lqr} = \frac{1}{20^2}$, $q_1 = 1$, and $q_2 = \frac{1}{20^2}$. This puts the highest cost on present TDE.

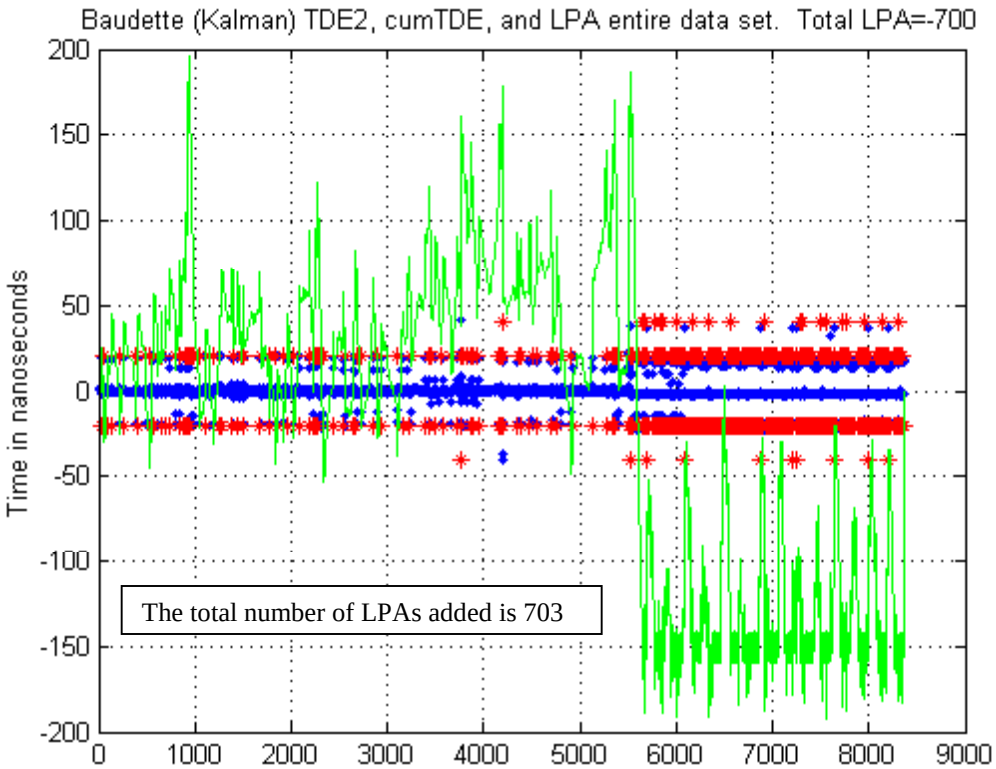


Figure 11

Notice that cumulative TDE and the number of LPA's do not meet specifications. Present TDE still varies greatly in the area of 20ns to –20ns.

Several weighting factors were tried. The best solution that was tried used the weightings of $R_{lqr} = 1,\ \mathrm{q}_1 = \frac{1}{500}$, and $\mathrm{q}_2 = \frac{1}{200}$. Figure 12 is Searchlight results.

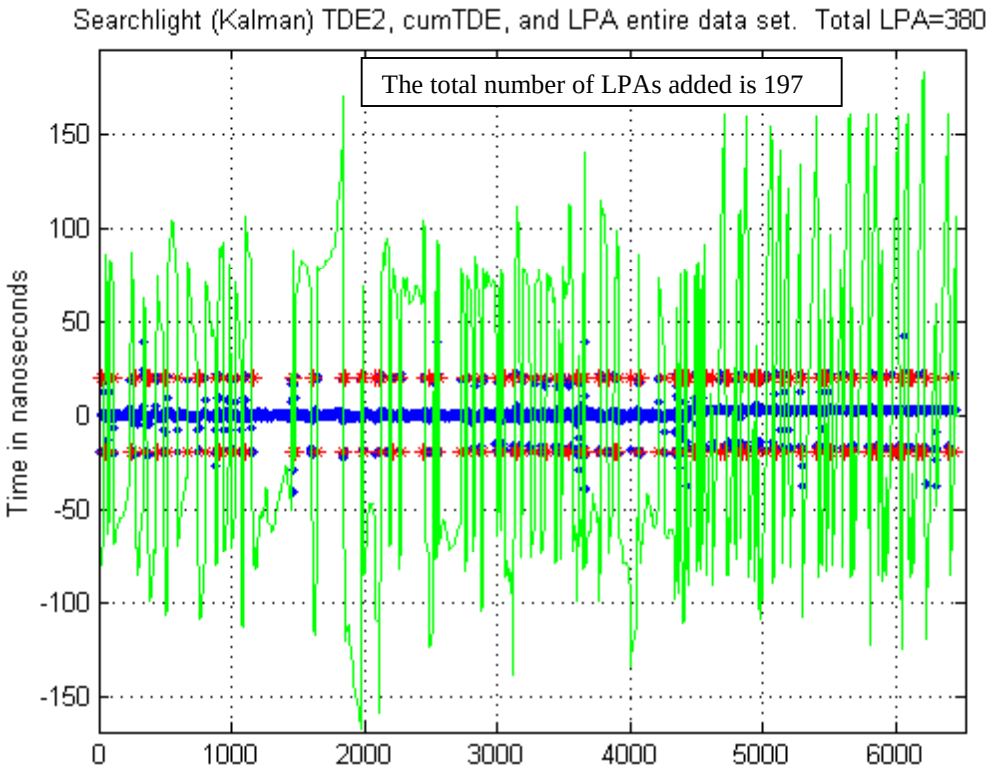


Figure 12

The number of LPA's added is slightly lower than the solution of LCCS and the present TDE has about the same response. Cumulative TDE varies between +150ns and –150ns. This is clearly non-optimal performance, but it is the best of the scale factors that were tested.

Cumulative TDE was plotted for Baudette Alpha-2 data (Figure 13).

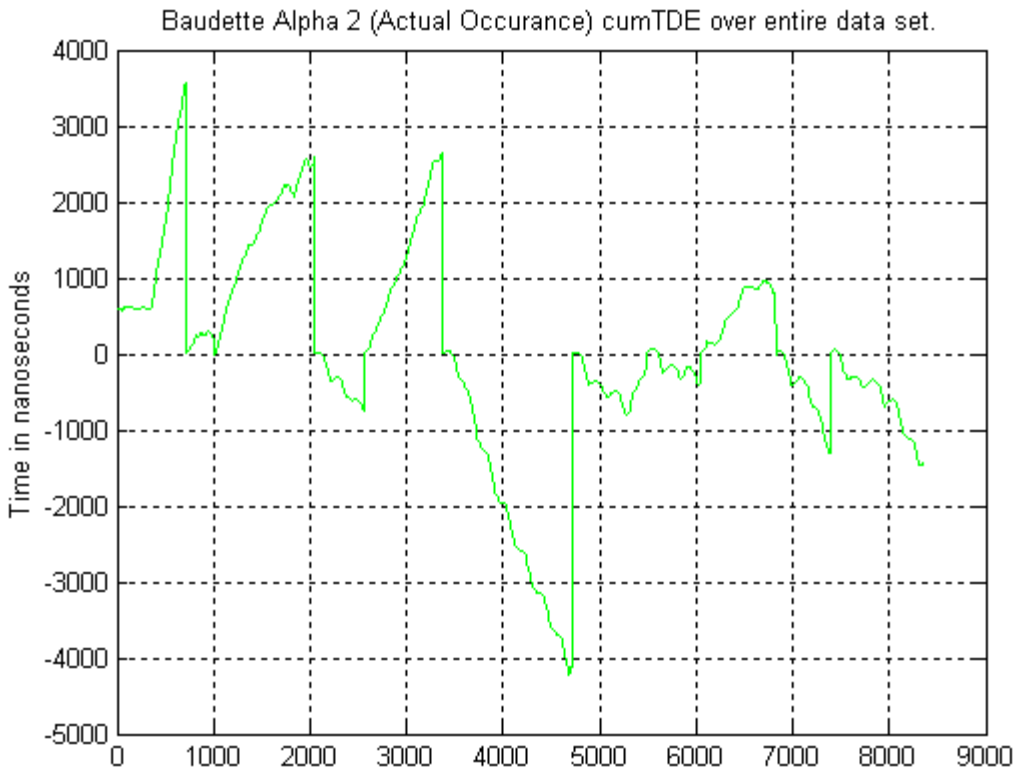


Figure 13

Notice that the value for cumulative TDE varies from 3500ns to –4000ns. This shows that we are only improving the Loran signal at the Alpha-1 monitoring site (the data being used to control).

Our calculations for cumulative TDE where checked with what LCCS was using. Figure 14 shows our calculation in magenta and LCCS's calculation in green.

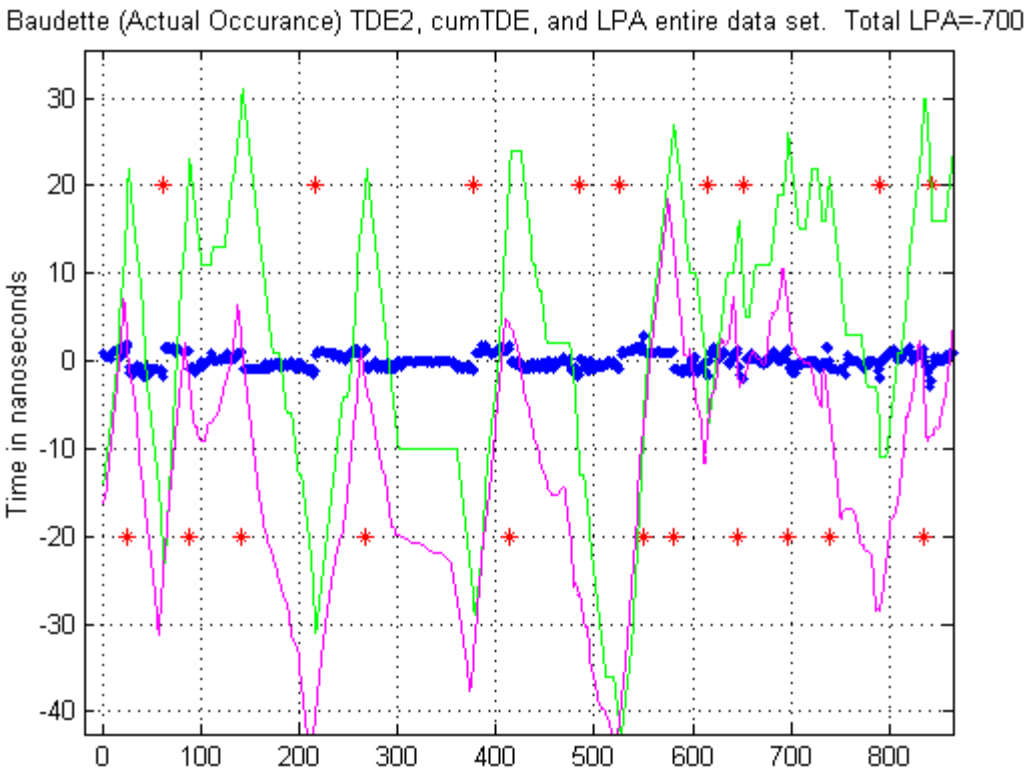


Figure 14

Notice that the two lines have the same shape but our calculation for cumulative TDE is of by as much as 22 nanoseconds.

Conclusions

As seen by the no control plots (Figures 3-4), Loran still needs to be controlled by some type of algorithm. The oscillators at these stations were replaced in September 2000. These new oscillators have steering functions. When they were installed the steering functions were set to zero. After looking at February and March 2001 data after LPA's were removed from the data, the steering functions should be adjusted accordingly to remove oscillator drift. The last few days of Baudette data with no control still do not meet the specification even after you remove the oscillator drift; thus, even with the steering functions tuned perfectly we will still need a time difference controller. This is not to say that we should not try and optimize the steering functions of the oscillators, because doing so will greatly reduce the number of LPA's added and therefore system performance.

The Kalman filter in it present form has not been able to keep present TDE closer than plus or minus 20ns. This is not to say that Kalman filtering cannot control present TDE. Present TDE performance using Kalman control is much better than using no control (were it drifts by hundreds of nanoseconds.

Cumulative TDE performance using the present form of the Kalman filter is not superior to LCCS's solution. When no control values are compared to values from the Kalman filter, the Kalman filter has far superior results.

Even with a perfect estimate of future TDE optimal LPA's will not be calculated unless the proper value for the L matrix is found. It appears that the discrete linear quadratic regulator was unable to do this. There are many different controllers that can be used in place of LQR such as H-2, H-∞, and L-1 norm minimization. H-2 minimizes the average, which may be good to control the cumulative TDE. H-∞ tries to minimize the worst possible case. This could be good to control the present TDE. The L-1 tries to minimize the number or size of elements, which may be a good technique for LPA control. A thorough investigation that relates modern control theory to the control policies should be conducted.

Developing new A, B, and H matrices to allow for both alpha-1 and alpha-2 data sets to be used in TD control. Alpha-2 data would have to have a small weight factor due to the monitoring station's "non-optimal" location. This could help improve over the present system due to what happens with the cumulative TDE at the Alpha-2 site. We do not want to just make Loran good in just one area at the cost of greatly decreasing performance elsewhere.

We were unable to find the optimal weighting factors to make system performance greatly better than LCCS. Further "tuning" of the Kalman filter needs to be completed. With further tuning and modification the Kalman filter paired with some type of optimal controller (i.e. LQR, H-2, H-∞, or L-1) for LPA recommendations, Loran system performance can greatly be increased.

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